Math 112 - Calculus II (Integral Calculus)

What is integral calculus?

Integral calculus is the study of how quantities accumulate, perhaps over time, distance or other variables. There are many applications:

- Suppose we know the intensity of sunlight on an area of land at various times throughout the day. How much solar power in total will be collected (accumulated) in one day? (Ecology/Climatology)

- What is the volume of liquid in a spherical tank, given its depth? (We accumulate cross-sectional areas to find volume) (Engineering)

- Suppose we know the concentration of a compound at various points in a solution. What is the total amount of compound in the solution? (How do the small amounts in each part of the solution accumulate to become the total? (Chemistry)

- Suppose we know how the birth and death rates of a population vary over time. What will be the net change in population over a given time period? (How do small changes in population accumulate to give the net change?) (Biology)

It is a surprising and very important result that these sorts of questions about accumulation are linked to the question studied in Calculus I about rates of change; the link is the Fundamental Theorem of Calculus, without doubt one of the most important discoveries in all of Western thought.
In Math 112 the fundamental tool developed to study accumulation is the definite integral; in symbols $\int_a^b f(x)\,dx$.

Example:
The answer in the first example above may be something like $\int_0^{24} 500\sin\left(\frac{\pi t}{24}\right)\,dt$ watt hours.

Once that tool is built, the Fundamental Theorem of Calculus is used to establish the link between it and the idea of derivative, studied in Math 111, Calculus I. This link allows one to calculate many definite integrals easily. With that in place, methods of constructing definite integrals to calculate various real-world quantities are studied.
Later in the course the idea of accumulation is used to study the notion of infinite series (sums) and their applications. What might it mean to add up infinitely many numbers? For example, what is

$$1+\frac{1}{4}+\frac{1}{9}+\frac{1}{16}+\frac{1}{25}+\cdots=?$$

(the sum goes on forever). It turns out one can make sense of infinite sums (the sum above is $\frac{\pi^2}{6}$!) and that there are many applications in science and technology.

In addition to its usefulness, and as for its counterpart Math 111, the ideas studied in Math 112 are basic to much advanced mathematics. Taking Math 111 and 112 is the first step in the study of university-level mathematics, and an interesting intellectual journey in its own right.

Prerequisites:
MATH 111 with a C or better.

Content:

- Definite Integral:
  1. brief review of derivatives and antiderivatives
  2. integration by substitution
  3. integration by parts
  4. other integration techniques, as time permits: trigonometric substitution, partial fractions use of tables
  5. numerical integration including Riemann sums, trapezoid and midpoint rules, Simpson's rule
  6. improper integrals

- Applications:
  1. constructing Riemann sums and evaluating integrals in a wide variety of settings, including area, volume, arc length
  2. applications from the natural and social sciences

- Differential Equations:
  1. slope fields
  2. Euler's method
  3. separating variables
  4. applications to growth and decay problems, including exponential, limited, and logistic models
  5. modelling other situations, as time permits

- Series:
  1. Taylor polynomials
  2. sequences and series
  3. Taylor series and applications
  4. error estimation

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